"Cloudiness: Note on a Novel Case of Frequency." By KARL PEARSON, M.A., F.R.S., University College, London. Received December 1,—Read December 16, 1897.

In a memoir on Skew Variation, contributed some time back to the 'Philosophical Transactions,'* I pointed out (p. 364) that we might expect theoretically to occasionally find U-shaped distributions of frequency. I was unable at that time to refer to any case actually known to me except Mr. Francis Galton's curve of "consumptivity." The data in that case did not seem to me sufficiently definite to base any elaborate calculations upon them. Quite recently, in studying Hugo Meyer's 'Anleitung zur Bearbeitung meteorologischer Beobachtungen für die Klimatologie,' Berlin, 1891, I came across, on S. 108, the table for the frequency of various degrees of cloudiness for the decade 1876–85, at Breslau. Although the method used for determining the extent of cloudiness is not entirely satisfactory, and, as Herr Meyer remarks, the observer must have had some personal bias with regard to the grade 9, still the observations are so numerous, and so markedly U-shaped, that I thought it well worth investigating how far my theory of skew variation would suffice to describe such a novel form of frequency.

The observations are as follows:—

Degrees of Cloudiness at Breslau, 1876—1885.

Degree	0	1	2	3	4	5	6	7	8	9	10
Frequency....	751	179	107	69	46	9	21	71	194	117	2089

The total number of days of observation is 3,653.

Clearly no cloudiness and absolute cloudiness are both maxima while the mean cloudiness will not be very far removed from minimum frequency.

The following data were obtained for the distribution by Miss Alice Lee, by the methods of the memoir referred to above:—

$$\text{Mean} = 6{\cdot}8292 \qquad \beta_1 = 0{\cdot}6112$$

$$\mu_2 = 18{\cdot}2999 \qquad \beta_2 = 1{\cdot}7414$$

$$\mu_3 = -61{\cdot}2030 \qquad 6+3\beta_1-2\beta_2 = 4{\cdot}3508$$

$$\mu_4 = 583{\cdot}1838$$

The theoretical curve is thus one of limited range.

* Series A, vol. 186, 1895.

Proceeding we found

$$\epsilon = 0{\cdot}00699 \qquad r = 0{\cdot}17958$$

$$m_1 = -0{\cdot}8774 \qquad a_1 = 4{\cdot}8109$$

$$m_2 = -0{\cdot}9430 \qquad a_2 = 5{\cdot}1705$$

The negative values of m_1 and m_2 show us that the theoretical curve has changed from its usual form to a U-shaped figure. The range given is $b = a_1 + a_2 = 9{\cdot}9814$, instead of the actual 10.

The distance d between mean and mode

$$= \frac{a_2 - a_1}{r} = 2{\cdot}0022.$$

Thus the start of the range is $4{\cdot}8270 - 4{\cdot}8109 = 0{\cdot}016$, instead of 0, and it runs to 9·998, instead of 10. We conclude accordingly that if the range of possible cloudiness had been quite unknown *à priori*, it would have been closely given by theory.

The modal frequency y_0 was found to be 50·7505.

Thus the theoretical equation to the frequency is—

$$y = 50{\cdot}7505\left(1 + \frac{x}{4{\cdot}8109}\right)^{-0{\cdot}8774}\left(1 - \frac{x}{5{\cdot}1705}\right)^{-0{\cdot}9430},$$

the origin being at 4·8270.

The modal value now corresponding to a minimum and not to a maximum as usual, the name "mode" ceases to be appropriate.* The observations and the above curve are given in the accompanying diagram, and it will be seen that there is a complete transformation of the usual frequency distribution to fit the altered state of affairs. With the asymptotic character of the curve, it is impossible to compare ordinates as giving the frequencies between 0 and 1, and 9 and 10. Accordingly the areas of the curve between 0·016 and 0·5, and between 9·5 and 9·998 were taken as the true measure of the frequencies of the degrees 0 and 10. These were obtained by means of the following formulæ :—

$A_1 =$

$$(y_0 b) \times \frac{n_1^{n_1}\, n_2^{n_2}}{(n_1 + n_2)^{n_1+n_2}} \times \left(\frac{x_1}{b}\right)^{1-n_1}\left\{\frac{1}{1-n_1} + \frac{n_2}{2-n_1}\left(\frac{x_1}{b}\right) + \frac{n_2\,(n_2+1)}{2\,(3-n_1)}\left(\frac{x_1}{b}\right)^2\right\},$$

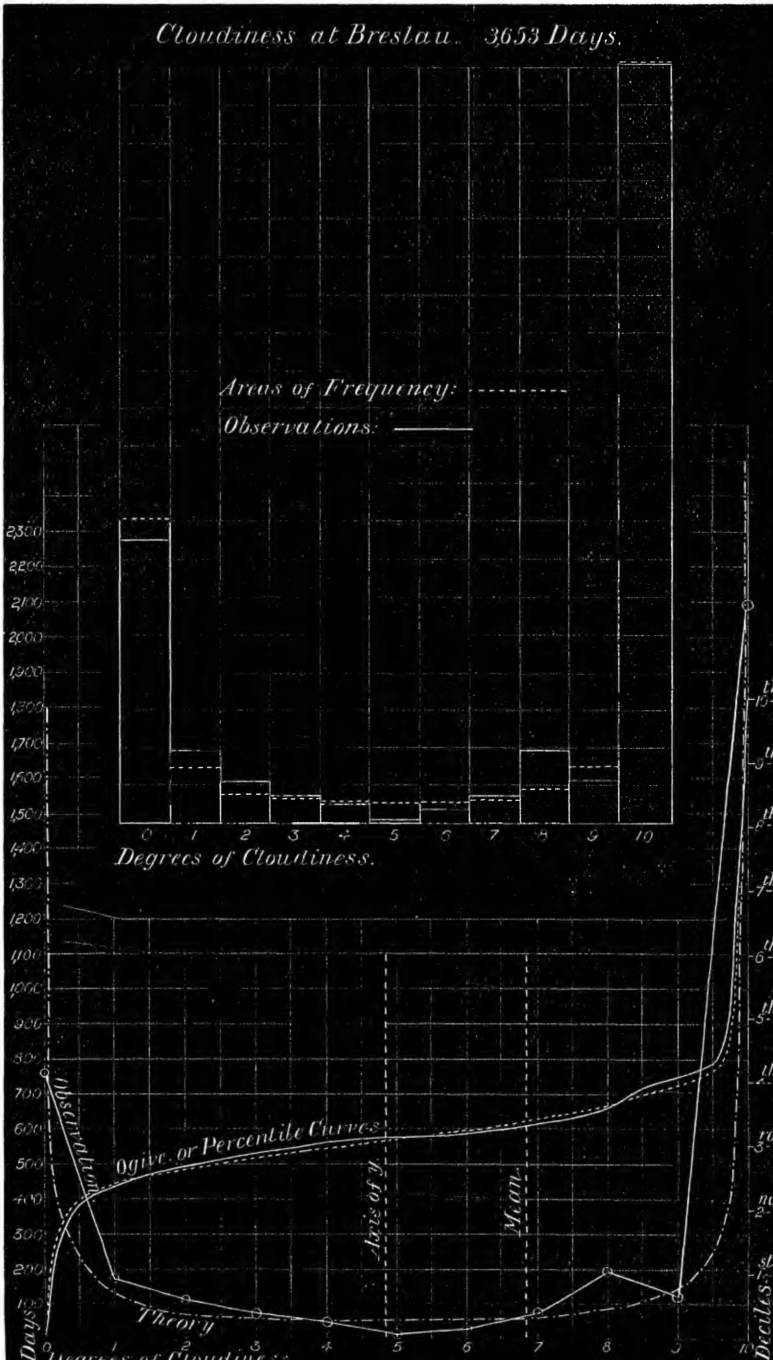
$A_2 =$

$$(y_0 b) \times \frac{n_1^{n_1}\, n_2^{n_2}}{(n_1 + n_2)^{n_1+n_2}} \times \left(\frac{x_2}{b}\right)^{1-n_2}\left\{\frac{1}{1-n_2} + \frac{n_1}{2-n_2}\left(\frac{x_2}{b}\right) + \frac{n_1\,(n_1+1)}{2\,(3-n_1)}\left(\frac{x_2}{b}\right)^2\right\},$$

where

$$n_1 = -m_1, \qquad n_2 = -m_2,$$

* The name *antimode* is now convenient.


Cloudiness at Breslau. 3653 Days.
Areas of Frequency:
Observations:
Degrees of Cloudiness.
Ogive, or Percentile Curves
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and A_1 and A_2 are respectively the areas of the curve measured for small lengths x_1 and x_2 at either end of the range.

The following gives a comparison of the frequencies of the various degrees of cloudiness, as given by observation, and by the areas of the curve:—

Degree.	Observation.	Calculation.
0	751	803
1	179	142
2	107	72
3	69	60
4	46	51
5	9	50
6	21	55
7	71	60
8	194	85
9	117	153
10	2089	2122

Considering the rough nature of cloudiness observations, the agreement must be considered fairly good, and very probably the smooth results of the theory* are closer to the real facts of the case than the irregular observations. The chief interest of this Note lies, however, in the fact that it shows the capacity of the theory of skew variation already developed to cover novel and unusual types of frequency.

"On the Occlusion of Hydrogen and Oxygen by Palladium."

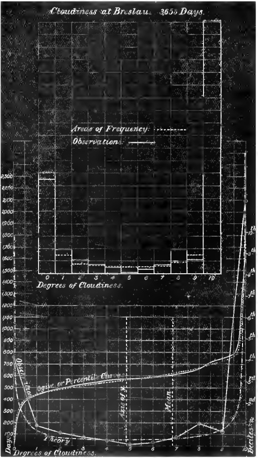
By Ludwig Mond, Ph.D., F.R.S., William Ramsay, Ph.D., LL.D., Sc.D., F.R.S., and John Shields, D.Sc., Ph.D. Received December 8,—Read December 16, 1897.

(Abstract.)

During their investigations on the nature of the occlusion of gases by finely divided metals, and in particular on the occlusion of hydrogen and oxygen by platinum black, the authors have had occasion to examine the behaviour of palladium to these gases.

The palladium was employed in three states of aggregation, viz., in the form of (*a*) black, (*b*) sponge, and (*c*) foil. Palladium black, prepared in the same way as platinum black, contains 1·65 per cent. of oxygen, or, taking the density of palladium black as 12·0, 138 volumes of oxygen. It differs from platinum black, however,

* The diagram on which the percentile curves are roughly drawn also indicates the amount of agreement by a histogram.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